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**B.Tech. Degree I & II Semester Examination in
Marine Engineering May 2019**

**MRE 1101 ENGINEERING MATHEMATICS I
(2013 Scheme)**

Time : 3 Hours

Maximum Marks : 100

(5 × 20 = 100)

- I. (a) Show that the function f defined as $f(x) = \begin{cases} x, & x \text{ is rational} \\ -x, & x \text{ is irrational} \end{cases}$ (6)
is not continuous at any point $x = 0$.
- (b) Evaluate $\lim_{x \rightarrow 1} \left[\frac{1 + \log x - x}{1 - 2x + x^2} \right]$ (7)
- (c) Find a root of the equation by Rolle's theorem $x \log x - 2 + x = 0$ lying in (1,2). (7)
- OR**
- II. (a) Find the evolute of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ (10)
(b) Find the n^{th} derivative of $y = x^2 \sin x$ at $x = 0$ (10)
- III. (a) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then show that $u_{xx} + 2u_{xy} + u_{yy} = \frac{-4}{(x+y)^2}$ (10)
(b) State and prove Euler's theorem for 3 variables. (10)
- OR**
- IV. (a) Find the Jacobean of $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$ (10)
(b) Find the percentage error in calculating the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when an error of +1% is made in increasing the major and minor axis. (10)
- V. (a) Find the condition for the straight line $lx + my + n = 0$ to touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (10)
(b) Derive the standard equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (10)
- OR**
- VI. (a) Find the equation of the hyperbola passing through the point (3,4) and has the straight line $2x + 3y - 5 = 0$ and $x - 2y + 1 = 0$ as asymptotes. (10)
(b) Show that the locus of the point of intersection of 2 perpendicular tangents to a parabola is its directrix. (10)

(P.T.O.)

VII. (a) Evaluate $\iint (x+y)^2 dx dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (10)

(b) Find the area of the curves $r^2 = a^2 \cos 2\theta$. (10)

OR

VIII. (a) Find the volume of the cylinder $x^2 + y^2 = ay$ bounded by the plane $z = 0, z = y$. (10)

(b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{x}{(1+x+y+z)^3} dx dy dz$. (10)

IX. (a) Prove that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal. Also obtain the scalar potential. (10)

(b) Find the divergence and curl of the vector $A = xyz i + 3x^2y j + (xz^2 - y^2z)k$ at the point $(2,1,2)$. (10)

OR

X. (a) Show that the volume of the tetrahedron ABCD is $\frac{1}{6} [AB, AC, AD]$. Hence find the volume of tetrahedron formed by the points $(1,1,1), (2,1,3), (3,2,2)$ and $(3,3,4)$. (10)

(b) Prove that $[(A+B+C) \times (B+C)]. C = [A B C]$ (10)
